which is very higher. Hence the next coordinate is to be calculated by considering the $x_{i+1}$ as $x_i$

$x_i = 2.086799548$

$x_{i-1} = 2$ & $x_{i-2} = 1$

$y_{i-1} = -1$, $y_{i-2} = -6$

$y_i = (2.08)^3 - 2\times 2.08 - 5$

$y_i = 8.98 - 4.32 - 5$

$y_i = 8.98 - 9.16$

$y_i = -0.17$

$$A = \frac{-1 + 0.17}{(2-2.08)(2-1)} + \frac{-6+0.17}{(1-2.08)(1-2)}$$

$$= \frac{-0.83}{-0.08\times 1} - \frac{5.83}{-1.08\times 1}$$

$$A = +\frac{0.83}{0.08} + \frac{5.83}{1.08}$$

$A = 10.37 - 5.3$

$A = 5.04$

$$B = \frac{-1 + 0.18}{1 - 2.08} - 5.08\,(2 - 2.08)$$

$$B = \frac{-0.82}{-1.08} - 5.08\times -0.08$$



$B = 10.37 + 0.40$

$B = 10.96986238$

$x_{i+1} = 2.09462409$

$\epsilon_i = 0.37355353\,\%$

$$x_{i+1} = 2.08 - \frac{2\times -0.17}{10.96 + \sqrt{120.12 - 4\times 5.04\times -\ldots}}$$

$$= 2.08 - \frac{0.34}{10.96 + 11.11}$$

$$= 2.08 - \frac{0.34}{22.07} = 2.08 - 0.015 = 2.1$$

$$\epsilon_i = \left|\frac{2.06 - 2.07}{2.06}\right| \times 100$$

## Quotient difference method

This is the general method which is used to calculate the different (approximate) root of algebraic eq$^n$. Let a given algebraic eq$^n$ is f(x)

$f(x) = a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 0$

where $a_0, a_1, a_2, a_3$ are the different polynomials & let $x_1, x_2, x_3$ be the roots of this eq$^n$ such that $0 < |x_1| < |x_2| < |x_3|$

then f(x) can be written as

$$\frac{1}{f(x)} = \sum_i a_i x_i \quad \text{--- (1)}$$

$$\frac{1}{f(z)} = \sum_{i} \alpha_i z_i \quad \text{--- (1)}$$

where $\alpha_i = \sum_{r=1}^{\infty} \frac{B_r}{z_{r+i}}$

Now two term is define known as Quotient $[q^{(i)}]$ & difference $(\Delta^{(i)})$ whose value can be expressed as

$$q_i^{(i)} = \frac{\alpha_i}{\alpha_{i-1}} \quad \text{--- (2)} \Big] \times$$

whose value can be expressed as

$$q_1^{(i)} = \frac{\alpha_i}{\alpha_{i-1}} \quad \text{--- (2)}$$

Mly the difference can be written as

$$\Delta_1^{(i)} = q_1^{(i+1)} - q_1^{(i)} \quad \text{--- (3)}$$

Now a relⁿ is obtained b/w the quotient & difference component which can be written as

for $\quad q_h \Delta_h^{(i)} = q_{h+1}^{(i)} = \Delta_h^{(i+1)} \; q_h^{(i+1)}$ --- (4)

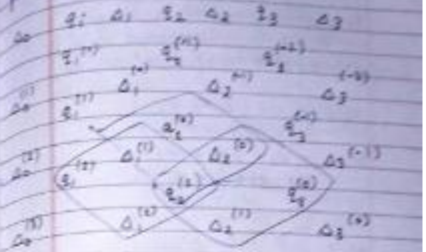
& for $\Delta_1^{(i)} + q_h^{(i)} = \Delta_{h-1}^{(i+1)} + q_1^{(i+1)}$ --- (5)

Also the sufficient condⁿ of this method is

$$\boxed{\Delta_0^{(i)} = \Delta_n^{(i)} = 0}$$



where n is the degree of the polynomial so the quotient difference table is given in the following manner



Ques Find the real roots of the eqⁿ $x^3 - 6x^2 +$ [illegible] $-6 = 0$

Solⁿ We have $x^3 - 6x^2 + 11x - 6 = 0$

Then the converted form of above eqⁿ can be written as

$$f(x) = -6x^3 + 11x^2 - 6x + 1 = 0 \quad \text{--- (1)}$$

Now from the condⁿ of quotient diff method we know that

$$\frac{1}{f(x)} = \sum_{i=0}^{\infty} \alpha_i x^i$$

Then

$$1 = f(x)\left[\alpha_0 x^0 + \alpha_1 x^1 + \alpha_2 x^2 + \alpha_3 x^3 + \cdots\right]$$

(14)

Then from eqⁿ no (1)

$1 = (-6x^3 + 11x^2 - 6x + 1)[\alpha_0 x^0 + \alpha_1 x^1 + \alpha_2 x^2 + \alpha_3 x^3 + \cdots]$

Now equating coe. of $x$ we get

$\alpha_0 = 1$ [component of $x^0$]

$\alpha_1 - 6\alpha_0 = 0$ [ " of $x^1$ ]

Then $\alpha_1 - 6 = 0 \Rightarrow \alpha_1 = 6$

also $11\alpha_0 - 6\alpha_1 + \alpha_2 = 0$

$11 \times 1 - 6 \times 6 + \alpha_2 = 0$

$\alpha_2 = 25$

IIIly.

$-6\alpha_0 + 11\alpha_1 - 6\alpha_2 + \alpha_3 = 0$

$-6 + 11 \times 6 - 6 \times 25 + \alpha_3 = 0$

$\alpha_3 = 6 - 66 + 150$

$\alpha_3 = 90$

Now the component of quotient is given by the formula

$q_1^{(i)} = \frac{\alpha_i}{\alpha_{i-1}}$

Then $q_1^{(1)} = \frac{\alpha_1}{\alpha_0} = \frac{6}{1} = 6$

$q_1^{(2)} = \frac{\alpha_2}{\alpha_1} = \frac{25}{6} = 4.16$

$q_1^{(3)} = \frac{\alpha_3}{\alpha_2} = \frac{90}{25} = 3.6$

Also the component of difference is given by the formula

$\Delta_1^{(i)} = q_1^{(i+1)} - q_1^{(i)}$

→ $\Delta_1^{(1)} = q_1^{2} - q_1^{1}$

$\Delta_1^{(1)} = 4.16 - 6$

$\boxed{\Delta_1^{(1)} = -1.84}$

→ $\Delta_1^{(2)} = q_1^{(3)} - q_1^{(2)}$

$\Delta_1^{(2)} = 3.6 - 4.16$

$\boxed{\Delta_1^{(2)} = -0.56}$

Now the quotient difference table is given by the table:

On considering the Rhombous in the table we get

$$\Delta_1^{(1)}$$

$$q_1^{(2)} \qquad q_2^{(1)}$$

$$\Delta_1^{(2)}$$

In this rhombus all the component is calculate except $q_2^{(1)}$ which is calculated by using formula

$$\Delta_h^{(i)} \, q_{h+1}^{(i)} = \Delta_h^{(i+1)} \, q_h^{(i+1)}$$

On replacing $i = h = 1$ we get

$$\Delta_1^{(1)} \, q_2^{(1)} = \Delta_1^{(2)} \, q_1^{(2)}$$

$$q_2^{(1)} = \frac{\Delta_1^{(2)} \, q_1^{(2)}}{\Delta_1^{(1)}}$$

$$\rightarrow \quad q_2^{(1)} = \frac{-0.56 \times 4.16}{-1.84}$$

$$q_2^{(1)} = 1.266$$

Ily on considering the another rhombous from the table we get

$$q_2^{(0)}$$

$$\Delta_1^{(1)} \qquad \Delta_2^{(0)}$$

$$q_2^{(1)}$$

In this all the quantities are known except $\Delta_2^{(0)}$ whose value can be calculated by considering the formula.



$$\Delta_h^{(i)} + q_h^{(i)} = \Delta_{h-1}^{(i+1)} + q_h^{(i+1)}$$

On replacing $h = 2$ & $i = 0$ we get

$$\Delta_2^{(0)} + q_2^{(0)} = \Delta_1^{(1)} + q_2^{(1)}$$

$$\Delta_2^{(0)} = -1.84 + 1.266 \qquad q_2^{(0)} = 0$$

$$\Delta_2^{(0)} = -0.54 \text{ Ans}$$

also the component of $\Delta_3^{(-1)} = 0$

Now the next another rhombous from table

$$\Delta_2^{(0)}$$

$$q_2^{(1)} \qquad q_3^{(0)}$$

$$\Delta_2^{(1)}$$

In this all the quantities are known except $\Delta_2^{(1)}$ whose value can be calculated by the formula

$$\Delta_h^{(i)} + q_h^{(i)} = \Delta_{h-1}^{(i+1)} + q_h^{(i+1)}$$

On replacing $h = 2$ & $i = 1$ we get

$$\Delta_2^{(1)} + q_2^{(1)} = \Delta_1^{(2)} + q_2^{(2)}$$

Q. Ramanjuan Method of Itteration

Shrinivash Ramanujan gave a iteration method to give the proper